

The Story of Rho

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■ **DEFINITION** We begin with the definition of Rho, the return efficiency. We define Rho as, $\rho = \mu/\sigma$, where μ is average monthly return, and σ is the standard deviation of monthly return. Rho measures “risk-adjusted” performance, i.e. return per unit of risk, and cannot be manipulated by increasing leverage. We use monthly data (instead of annual data) since that is the time frame most often used to present/report CTA/HF returns in the industry. Rho can be conveniently visualized as equivalent to a Sharpe ratio with risk free rate of zero. In other words, Rho captures the essential idea behind Sharpe ratio in a more convenient and useful manner.

■ **HISTORY** The concept of Rho was invented approximately 10 years ago. It was designed to overcome some of the limitations of the Sharpe ratio. Specifically, it was simpler to calculate, easier to interpret, and could be used to directly connect the expected returns and drawdown risk inherent in a CTA track record. It also allowed an “apples-to-apples” comparison between CTAs or HFs.

At that time, the popular measure of risk-adjusted performance was the Sharpe Ratio, which uses annualized performance, whereas most CTA data are presented on a monthly basis. Most importantly, Sharpe ratio cannot be related to actual draw downs on the track record. The Sharpe ratio is difficult to calculate, because it needs historical data on risk-free rate. Also, negative values of the Sharpe ratio cannot be easily interpreted.

Since Rho was designed to differentiate between CTAs, it can also be used to differentiate between trading systems during the design phase, since CTA track records ultimately represent trading systems. It turns out Rho is an excellent “all-in” or overall measure of system performance, that takes into account both the amount of return (average) as well as its consistency (standard deviation).

It is convenient to use the mean (μ) and standard deviation (σ) of CTA monthly returns because the field of statistics uses these quantities to describe continuous probability distributions, such as the normal distribution, the so-called “bell” curve. We can then visualize CTA monthly performance as belonging to some “normal” distribution, and the return for any particular month is merely a random sample from this distribution.

■ **THE NORMAL DISTRIBUTION AND Z-SCORE** We know the Normal distribution, which is the bell curve with some mean (μ) and standard deviation (σ). There is also the standard normal distribution with mean = 0 and standard deviation = 1. The way to convert from the normal distribution to the standard normal distribution is through the Z-score. For any value x under the normal distribution,

$$Z\text{-score} = (x - \mu) / \sigma.$$

Now suppose we did not know the mean return, but assumed that it was at least equal to the risk free rate. Then the Z-score begins to look a lot like the Sharpe Ratio discussed below, where x is the expected return, and σ is the known volatility of the program.

■ **RHO AVOIDS MANIPULATION OF SHARPE RATIO BY INCREASING LEVERAGE** By way of introduction, the definition of Sharpe Ratio (SR) = $(A - R) / S$, where A is the expected annualized return, R is the annualized risk free rate, and S is the annualized standard deviation (or volatility). A higher value of SR is preferred to a lower value of the SR and is often used to compare two CTAs. However, this ratio can be manipulated using leverage. We note that increasing leverage always increases the standard deviation of monthly returns, since the difference between the average return and the highest or lowest return also increases.

Now consider the definition of Sharpe Ratio = $(A - R) / S$, where A is the expected annualized return, R is the annualized risk free rate, and S is the annualized standard deviation (or volatility). Say $A=16\%$, $R=3\%$, and $S = 12\%$, gives $SR = (16-3)/12 = 13/12 = 1.08$. Now say we increase leverage (and therefore return) by 25% in the above example. Then $A=16*1.25=20\%$, $S=12*1.25 = 15\%$, and the new $SR = (20-3)/15 = 17/15 = 1.13$! Since a “higher” value of Sharpe is often preferred to a “lower” value of the Sharpe ratio, one can manipulate the SR by increasing leverage by increasing risk! This is precisely the opposite of what most people expect when they pick a “higher” value manager. Hence, it is dangerous to compare two managers on the basis of Sharpe ratio alone.

Now consider the case when $R = 0$. Now the low leverage Sharpe = $(16-0)/12 = 1.33$; with higher leverage $SR = (20-0)/15 = 1.33$! Thus, we can filter out the effect of increasing or changing leverage only if the risk free rate, $R = 0$. Since Sharpe ratio does not use a $R=0$, it can give misleading information. On the contrary, by definition, the Rho uses a risk free rate = 0, and hence it is insensitive to changes/differences in leverage. Hence, using Rho to compare two track records is inherently less risky than using the Sharpe ratio. Hence Rho is a better, more robust way to compare two track records (or trading systems), on an “apples-to-apples” basis.

■ **RELATIONSHIP BETWEEN RHO AND SHARPE RATIO** We can now show the connection between Rho and Sharpe Ratio. Rho can be visualized as the equivalent of a Sharpe ratio with risk free rate of zero.

$$\begin{aligned} SR &= (A-R)/S; \text{ if } R=0, \text{ then } SR = A/S. \\ SR &= A/S = (12\mu)/(3.4641\sigma) = 3.4641 (\mu/\sigma) = 3.4641 (\rho) \\ SR &= 3.4641 * \rho \text{ (with } R=0) \end{aligned}$$

Hence, Rho can be visualized as equivalent (but not equal) the Sharpe Ratio with risk free rate = zero. When we compare two CTAs or HFs, the constant 3.4641 (= square root 12) will be cancelled out. Hence, we only need to calculate Rho to compare systems/ CTAs/ HFs.

■ **IMPLICATIONS FOR SYSTEM DESIGN** We know, from its definition of $\rho = \mu/\sigma$, that ρ can be increased by

1. increasing μ for the same level of, (i.e. larger numerator, fixed denominator),
2. or decreasing the standard deviation for the same average return μ (i.e. fixed numerator, but smaller denominator)
3. doing both. The implication of a higher Rho for system design is that the package accomplishes one of the aforementioned goals. Hence a system with a higher Rho had better systems (higher μ), or used lower leverage or reduced the risk due to correlation offsets in the portfolio (lower σ), or did both. Since Rho cannot be manipulated by increasing leverage, it gives a more accurate measure of overall effects system design and portfolio selection.

■ **RHO CONNECTS RETURNS AND EXPECTED DRAWDOWNS** Our proprietary research using the returns of >100 CTAs and > 50 Hedge Funds shows that the maximum drawdown Max DD is approximately equal to $3\sim 4 * \sigma$, the standard deviation of monthly returns.

$$\begin{aligned} \text{Max DD } \Delta &= (3\sim 4) * \sigma. \\ &\text{(we will use } 4\sigma \text{ for our work).} \end{aligned}$$

This is a very important relationship with great practical significance, because it can be used to accurately “prepare” a customer for expected drawdowns. As a CTA, you can set up a variety of programs with different drawdown targets.

Say a client says they have a risk comfort level of 20%, and they are “aggressive” investors. You can then use $20/3 = 6.67\%$ as the target for their monthly standard deviation. Next, we can estimate the expected monthly return. Since, $\rho = \mu/\sigma$, $\mu = \rho * \sigma$.

Expected Annual return =
 $12 * \mu = 12 * \rho * \sigma = 12 * \rho * (\Delta/4) = 3 * \rho * \Delta$

This equation shows that a higher Rho is valuable because the client will make more money for the chosen level of risk. So we can directly connect the risk-preference of the client, to the performance of the CTA to predict what the client can expect (over the long-run). (see table 1) So Table-1 clearly shows why a higher Rho is preferred to a smaller one, since it means the client can make more money for the same level of expected draw-downs.

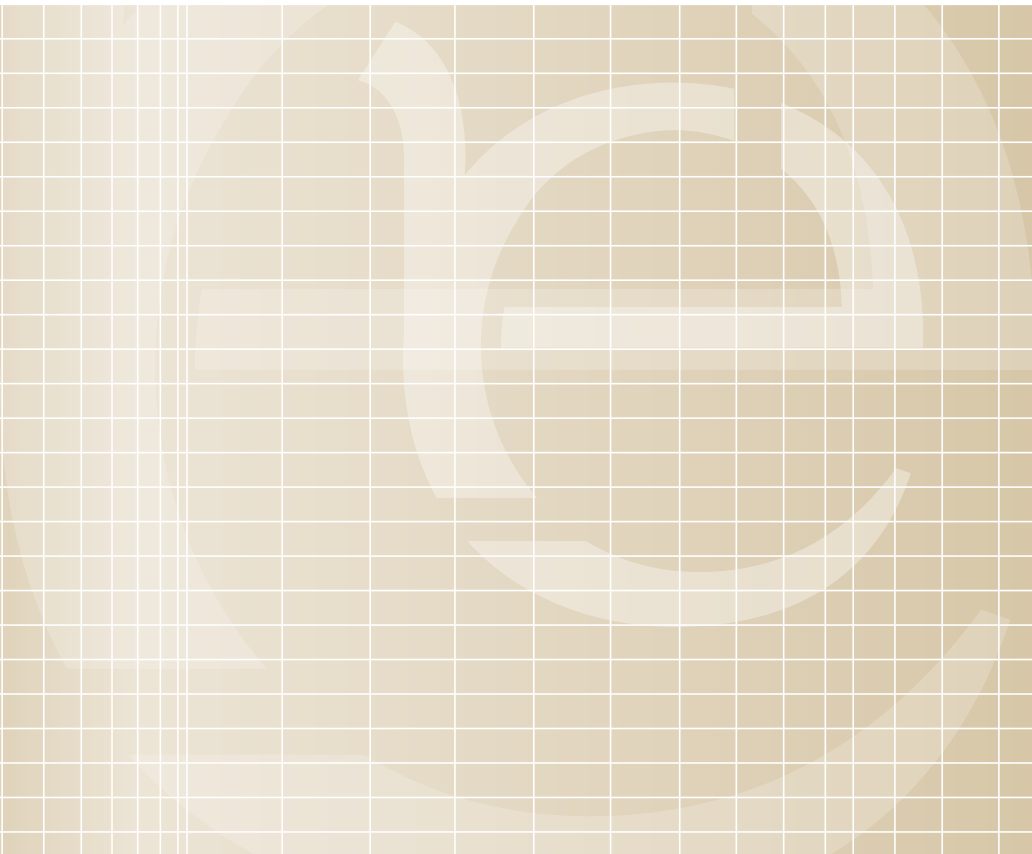
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Table-1	Aggressive Client	Conservative Client
Drawdown target level	20%	20%
Monthly Vol Multiplier	20 = 3*σ	20 = 4*σ
Monthly Std dev σ (%)	6.67%	5%
Return Efficiency ρ	0.35	0.35
Expcted Monthly Return (%)	2.33%	1.75%
Expcted Annual Return (%)	28%	21%

■ **RHO SIMPLIFIES COMPARISON BETWEEN CTAS** Each CTA uses a different amount of overall leverage. Most CTAs or HFs have many systems, and they may take different amounts of initial risk for each system per market. Besides, the level of leverage used may vary over time (if they are making money or losing money). Hence, we cannot say one CTA or HF is better than another because it made more money, without allowing for leverage. Since Rho is insensitive to leverage, it can be used to make an “apples-to-apples” comparison across different time periods and investment styles.

■ **RHO IS SIMILAR TO OTHER STATISTICAL MEASURES** Rho can be related to the signal-to-noise ratio, statistical distributions such as t-distribution or the Z-score for normal distributions. The Z-score = $(y-\mu)/\sigma$, where y is some value under the normal distribution. You can see that the Z-score is very similar to the Sharpe ratio.

■ **SUMMARY** Rho is simpler, more robust and more useful than Sharpe Ratio to evaluate CTA/HF/System performance. It directly connects risk borne by the investor to the returns generated by the manager. Hence, it is a measure of return efficiency, analogous to miles/gallon for a car or “bang for the buck” for defense expenditures.



Rho Asset Management (“Rho”) is a Swiss based long/short directional futures investment manager (CTA) established in 2007. The founders of Rho have combined more than 50 years of experience in alternative investments and a successful track record in trading and managing client assets.

Our goal is to provide the highest possible RETURN EFFICIENCY to our investors using ALTIUS, CITIUS and FORTIUS, all fully automated trading programs. Rho is research and technology driven, specializing in design and implementation of systematic trading strategies. All strategies are based on quantitative analyses of price behavior in the global financial and commodity markets.

Rho is dedicated to providing our clients with tangible value in investment performance and quality of client service.